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Estimates of glacier equilibrium line altitudes by the Area × Altitude, the Area × Altitude Balance Ratio and the Area × Altitude Balance Index methods and their validation☆

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Abstract

Unlike other methods of estimating the Equilibrium Line Altitude of present or former glaciers from morphometric data (as distinct from direct observations of the glacier mass balance), the Area × Altitude, the Area × Altitude Balance Ratio and the Area × Altitude Balance Index methods take explicit account of hypsometric differences between glaciers and thus yield more reliable results. In addition they offer the means of applying various mass balance/altitude relationships of increasing complexity and examining which of these is most correct; the last of these methods is newly developed to permit the application of any desired relationship. Their general adoption has been restricted hitherto by computational problems, but this objection is removed by the easy-to-use spread sheets presented in this paper. By whatever method estimates are derived, it is essential to validate the optional variables used in the computations and methods for doing this are set out.

1. Introduction

It is often required to estimate the equilibrium line altitude (ELA) of a present glacier for which no direct mass balance observations are available, or to estimate the ELA of a former glacier of which only its extent can be inferred from geomorphological features. Unless otherwise stated, in the following discussion the term ELA will refer to a glacier in a hypothetical balanced state, i.e. when the net mass balance of the whole glacier is or would be zero. On an advancing or retreating glacier this is not at the position (termed 'current ELA' below) where surface balance measurements or aerial observations of the firnline would show equality of local accumulation and ablation (Benn and Lehmkuhl, 2000).

The Area × Altitude (AA), Area × Altitude Balance-Ratio (AABR) and the Area × Altitude Balance Index (AABI) methods of doing this take account of the hypsometry (the detailed distribution of surface area with respect to altitude) of a glacier, unlike other methods in general use (Table 1). All are based on the principle that parts of a glacier which are far above or below the ELA have greater spot net balances (plus or minus) and so have more influence on the total mass balance of a glacier, and hence on the ELA, than those which are close. Thus they require knowledge of the position of the margin of a glacier and contour data for its surface, so that the area and mean altitude of successive contour belts of its surface can be determined. For former glaciers there are necessarily some uncertainties about both these variables, especially where there is a large steep headwall (Benn and Lehmkuhl, 2000) but these are no worse than those which affect other morphometric methods. All three are quite distinct from the rather similarly named Accumulation Area Ratio (AAR) method which takes no explicit account of the detailed hypsometry or the balance ratio (BR).

☆These spreadsheets can be down-loaded from the author or from: www.ggy.bris.ac.uk\research\bridge\resources\resources.htm <http://www.ggy.bris.ac.uk/research/bridge/resources/resources.htm>

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Table 1
Comparison of the ratio and index methods of ELA estimation

	THAR	AAR	AA	AABR	AABI
Data needed					
Toe and headwall alts.	+				
Glacier outline		+	+	+	+
Glacier contours		(+)[a]	+	+	+
Information applied to the estimate					
Altitude range	+		(+)	(+)	(+)
Broad hypsometry		+			
Detailed hypsometry			+	+	+
Balance ratio				+	
Detailed balance index					+

[a]In principle this method only requires a surveyed line of altitudes up the centre line of the glacier, and a division of the surface area in the selected ratio(s), but it is usually done by assessing contour belt areas.

The AA method makes the assumption that the BR (the ratio of the slopes of the mass balance/altitude graph below and above the ELA) is unity and in consequence it is straight-forward to calculate. The AABR method assumes that the mass balance/altitude curve consists approximately of two linear segments, usually with different slopes above and below the ELA, and adds the refinement of applying any desired BR to the estimate, so the calculation is more complex. The AABI method develops this further for the application of *any* predetermined form of mass balance curve. This last method has not previously been published.

The validation of estimates of ELAs made by these and other methods seldom receives sufficient attention. The choice of the ratio or index used is often justified by reference to other glaciers which may be in an entirely different climatic environment. With former glaciers, which are themselves being used to try to infer past climate, the problem is especially acute. The large number of occasions on which the useful study of Meierding (1982) of ELAs in the Colorado Front Range has been cited to justify the use of the same ratios elsewhere and at other times is lamentable. However, a statistical approach offers a practical alternative.

2. The Area × Altitude method

This method was devised by Kurowski (1891), who used it to estimate the ELAs of Alpine glaciers. It was subsequently adopted by other workers such as Drygalski (1942), but its wider application was restricted by the lack of reliable contoured maps of glacierised or glaciated regions. It involved taking a trial value for the ELA (e.g. the mid-altitude), then multiplying the areas of successive contour belts by their mean altitude difference above (+) or below (–) the trial ELA. The algebraic sum of these indicates whether the trial ELA should be moved up or down, and the calculation is reiterated until a zero sum is obtained.

This calculation was tedious when done manually and the method fell out of use until, with the advent of computers, it was used in an ALGOL programme (later rewritten in Visual Basic) by Osmaston (1965, 1975, 1989a, b) and Kaser and Osmaston (2002) to estimate the ELAs of present and former glaciers on East African mountains. Osmaston also developed it further into the AABR method.

Sissons (1974, 1980) rediscovered the AA method, applied it to former Loch Lomond Advance glaciers in Scotland and the English Lake District. He greatly simplified the calculation by the insight that the ELA it yielded is the median value of the product of the area and mean altitude of successive contour belts of the glacier surface (not the median altitude which is the centre value of the altitude range). This can be calculated easily and quickly by summing the products of successive contour belt areas and mean altitudes, then dividing this sum by the total area. Although the assumption of linearity is seldom quite correct, this method gives more reliable ELA estimates than AAR or THAR (Toe-Headwall Area Ratio) where the glacier is not of 'simple standard valley glacier' form, e.g. piedmont or plateau or with multiple tributaries.

3. The Area × Altitude Balance Ratio method

Like the AA method, this is based on the principle of weighting the mass balance in areas far above or below the ELA by more than in those close to it. However this is then refined by providing for different linear slopes of the mass balance/altitude curve above and below the ELA. Many glaciers conform roughly to this specification, and it serves as a useful first approximation for former glaciers for which there is no a priori knowledge about their mass balance. It was developed by Osmaston who originally termed it the Area-Height-Accumulation method (Osmaston, 1965, 1975, 1989a, b; Kaser and Osmaston, 2002) for use on East African mountain glaciers. Its operation was tested by Furbish and Andrews (1984) who termed it the BR method, on existing valley glaciers in Alaska where they found it gave good ELA estimates. Benn and Gemmell (1997) provided a spread sheet available on line for calculating ELAs by this method, but unfortunately the website concerned is no longer functioning and the programme is now only available from the authors. The programme was well presented, easy to use and apparently functional. Unfortunately it gave inaccurate results when tested with simple trial data and, because it was in a complex format using both MS Excel and MS Visual Basic with a packet Search programme, it was difficult to debug.

The programme presented here has been deliberately designed to be as transparent as possible, presented clearly stage by stage and with the formulas controlling each stage clearly available for examination (and if desired for modification). This means the programme is rather long, but experienced programmers will be in a position to improve and compact it easily. Appendix A contains clear instructions for compiling the spread sheet.

The programme is in two parts. The first is a calculation of the ELA by the short AA method; this gives a good preliminary estimate of the ELA and is so simple that is unlikely that any error will occur in it. Thus it can be used as a general check on results, and in particular it should be equal to the AABR when $BR = 1$. An automatic internal check is provided using this relationship. The second part estimates the ELA by the full iteration procedure using a series of trial contour altitudes for the ELA and for each calculating the net balance of the whole glacier. These results are then reviewed and the pair at which the balance changes sign is selected. The exact ELA between these values is then estimated by the proportions of the two net balances. The detailed construction of the spread sheet is described in Appendix A, and Table 2 shows the appearance of the spread sheet with the initial AA calculation and the first iteration of the balance calculation with the first trial contour. The programme may then be rerun after inserting another value for the BR.

4. Procedure for AABR

1. Check correct operation of spread sheet with trial data.
2. Check that contour table will cover glaciers to be examined and that VI is correct.
3. Enter contour belt area table for glacier 1.
4. Enter altitude of first trial reference contour.
5. Enter $BR = 1$ and check correct operation of programme.
6. Record ELA.
7. Enter in succession a series of BR values (e.g. 1, 1.5, 2.0, 2.5, 3.0, 3.5) and record the ELA for each. Ratios can be selected by a priori knowledge of what is likely; most glaciers are likely to have BRs of 1.5–3.5, though on a debris-covered one it may be less than 1.
8. Repeat for the other glaciers.
9. Enter results in a spread sheet for displaying them and calculating the mean and standard deviation of the estimated ELAs for each BR value.
10. Select the BR with the lowest standard deviation, which indicates the ELA with the best statistical probability of being correct.
11. Plot the ELAs on a map to see if they show any pattern of grouping, clines or sloping surfaces and re-analyse the data accordingly.

5. The Altitude × Area Balance Index method

This new method permits the application of any desired mass balance/altitude curve to any glacier for which the areas of its surface contour belts can be estimated. The Balance Index is a non-dimensional number representing the relative value of the mass balance at each altitude. This may be equal to the balance in absolute terms (e.g. metres water equivalent) where this is known from an already studied reference glacier, or it may be the ratio of the balance at all other points to one particular arbitrarily selected point on an idealised form of mass balance/altitude curve.

Fig. 1 shows how these values may be taken from a generalised mass balance curve for the Khumbu Glacier (Inoue, 1977). In this case the absolute values of the mass balance are available, but for demonstration and simplification purposes the value of the unchanging segment in most of the accumulation area has been taken as the reference value, unity.

The method of calculation is similar to that for the AABR method in that glacier balance is tested for a succession of trial ELAs positioned at whole number contours. For each contour belt, the product of its area and altitude difference from the trial ELA is multiplied by the appropriate BI taken from a look-up table. These are summed to give a balance for the whole glacier. When two adjoining trial ELAs are identified which give respectively a positive and negative mass balance, then the ELA is between these at an intermediate altitude determined by the ratio of the two balances. A simple spread sheet for doing the calculation for one trial ELA is given in Appendix B. This may either be used for the manual application of a series of trial contour ELAs, the results of which can easily be used to pinpoint the final value; or it may be inserted into a fuller programme like that given for AABR for automatically reiterating and

Table 2
The main part of an AABR spread-sheet showing a set of test data in undefined units of measurement

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Glacier number or name	Contour vertical interval (VI)	Contours incl. all crossing glacier	Mean belt altitude	Contour belt area	Mean altitude × Area	Balance Ratio (BR)	ELA trial contour (1)	Belt area × Alt above trial contour (1)	Area × Alt × Balance Ratio for contour(1)	ELA trial contour (2)	Belt area × Alt above trial contour(2)	Area × Alt × BR for contour (2)	ELA trial contour (3)
2	Check	100.0	0	50		0	2.0	500	0	0	600	0	0	700
3	Model 2		100	150		0			0	0		0	0	
4			200	250	10	2500			−2500	−5000		−3500	−7000	
5			300	350	20	7000			−3000	−6000		−5000	−10 000	
6			400	450	30	13 500			−1500	−3000		−4500	−9000	
7			500	550	40	22 000			2000	2000		−2000	−4000	
8			600	650	50	32 500			7500	7500		2500	2500	
9			700	750	30	22 500			7500	7500		4500	4500	
10			800	850	10	8500			3500	3500		2500	2500	
11			900	950		0			0	0		0	0	
12			1000	1050		0			0	0		0	0	
13			1100	1150		0			0	0		0	0	
14			1200	1250		0			0	0		0	0	
15			1300	1350		0			0	0		0	0	
16			1400	1450		0			0	0		0	0	
17			1500	1550		0			0	0		0	0	
18			1600	1650		0			0	0		0	0	
19			1700	1750		0			0	0		0	0	
20			1800	1850		0			0	0		0	0	
21			1900	1950		0			0	0		0	0	
22														
23	Totals				190	108 500			13 500	6500		−5500	−20 500	
24														
25	AA ELA (median alt × area, shortcut method assumes BR = 1) =					571	CHECK	TRUE						
26	AABR ELA for BR = 1, first figure in this row (interpolated between contours) =											571		
27	AABR ELA for other BRs, first figure in this row (interpolated between contours) =												524	
28														
29				Results										
30				*Method*					*Bal. Ratio*			*ELA*		
31				AA					1			571		
32				AABR					1			571		
33				AABR					2			524		

A trial ELA of 500 has been applied and a Balance Ratio of 2.0. The internal check indicates that the programme is working correctly, displaying 'TRUE' and yielding two identical ELA estimates by long- and short-cut methods of 571 assuming BR = 1, and a third estimate of 524 for BR = 2. The full table extends to column AK, repeating columns K, L, M for successive values of trial ELA, until the columns AI, AJ and AK are reached for trial number 10.

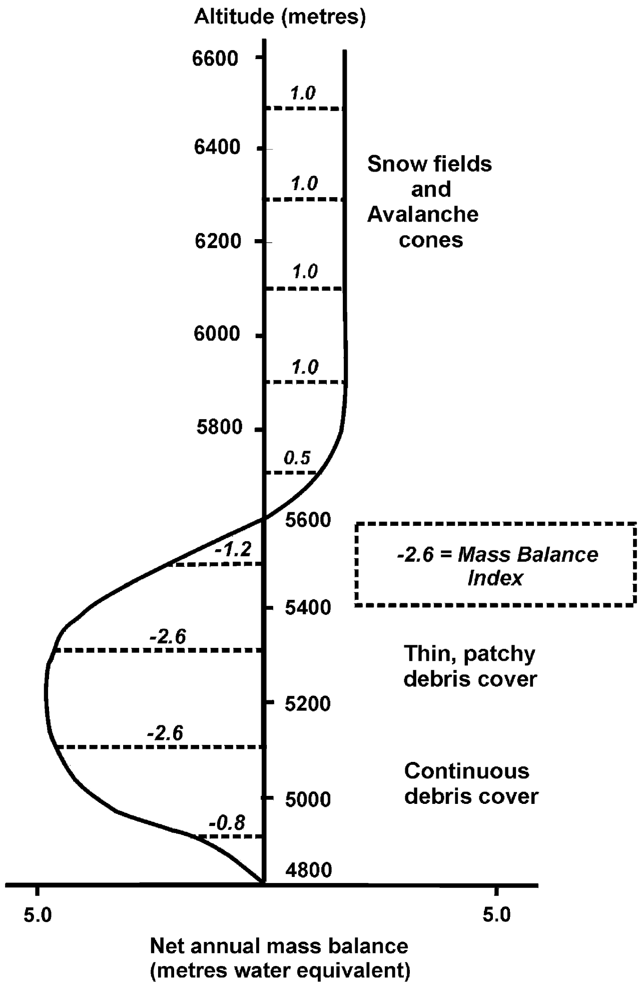

Fig. 1. Khumbu Glacier, Nepal. Generalised mass balance curve after Inoue (1977) and Benn and Lehmkuhl (2000).

testing a series of ELAs, and then evaluating the final answer.

Table 3 shows a sample output based on the balance indices of the Khumbu Glacier and a trial ELA reference contour of 5200 m. The positive residual of 83 shows that there is a slight excess of accumulation so this trial contour is too low. Substitution of a trial contour value of 5400 m gives a negative residual of −64 showing that there is now an excess of ablation, so this trial contour is too high. Thus the precise steady state ELA is at

$$5200 + 200(83/(83 + 64)) = 5313\,\text{m}.$$

This is lower than the observed current ELA of the Khumbu Glacier, but the contour belt areas are not intended to represent the Khumbu Glacier, being some purely hypothetical values representing some other similar glacier. However by inserting a contour belt area table for the Khumbu Glacier it would be possible to check whether its hypothetical steady state ELA is similarly below the current ELA as would be expected on a retreating glacier.

There are indications that many former glaciers may have had debris-covered tongues and this method offers an opportunity to test whether a balance index curve that reflects this characteristic, like that of the Khumbu Glacier (Fig. 1), gives an improved (i.e. lower standard deviation) ELA estimate for a group.

6. Glacier area and condition

For a former glacier, the areas of contour belts should in principle be based on a hypothetical reconstruction of the ice surface, with the contours appropriately adjusted. Sissons (1974, 1980) did this for the very small glaciers with which he was dealing, but the procedure is laborious and uncertain on larger glaciers. Instead the areas should be assessed from ground contours, the ELA estimated and its position located on the map. The thickness of the glacier should be estimated at this point from a straight line joining valley-side trim lines or the crest-lines of lateral moraines and then added to the calculated ELA.

The glacier characteristic that directly reflects the current climate is the current ELA. When the ELA of a former glacier is estimated morphometrically using the position of large terminal moraines one may be reasonably confident that this represented, at least for a short time, a 'quasi-balanced state' or steady state at which the balanced state ELA was the same as the current ELA and thus had a definite climatic significance. However, a retreating glacier has a greater extent than would be in equilibrium with its actual current ELA. Thus the rise in its hypothetical balanced ELA lags behind and is less than that of its current ELA. The opposite occurs with advancing glaciers. Thus it is difficult to know what significance should be attached to a hypothetical balanced state ELA estimated from morphometric data (whether by THAR, AAR, AA, AABR or AABI) on a glacier that is known to be retreating, as so many tropical glaciers now are. The only reliable conclusion is that the current ELA is higher than that indicated by morphometric methods by an unknown amount. On some tropical mountain glaciers the current ELA appears to be higher than the top of the glacier, so that the glacier is entirely in the ablation zone. This lag is greater with large glaciers than with small ones. This means that former ELAs should preferably be estimated from groups composed of medium sized to small glaciers, excluding the largest (where ablation on long debris-covered tongues is also difficult to quantify) and perhaps also the smallest which are more subject to the effects of local relief.

In a large mountain range there will be a significant variation in the ELA over its area, so large trunk

Table 3
AABI spread sheet for Khumbu Glacier, Nepal, using Balance Indices derived from Fig. 1

MODEL GLACIER Mean belt alts.above or below ELA	MODEL GLACIER Balance index (BI) for each belt above or below ELA	NEW GLACIER Contour belt area	NEW GLACIER Mean belt altitude	ELA trial reference contour	Mean belt Alt MINUS Ref. contour	Mass Balance index (BI)	Belt Area × BI
−1100	−0.8	10	4900	5400	−500	−2.6	−26
−900	−0.8	30	5100		−300	−2.6	−78
−700	−0.8	50	5300		−100	−1.2	−60
−500	−2.6	40	5500		100	0.5	20
−300	−2.6	30	5700		300	1.0	30
−100	−1.2	20	5900		500	1.0	20
100	0.5	10	6100		700	1.0	10
300	1.0	10	6300		900	1.0	10
500	1.0	10	6500		1100	1.0	10
700	1.0					0.0	0
900	1.0					0.0	0
1100	1.0					0.0	0
1300	1.0					0.0	0
						SUM =	−64

This example covers only one trial reference contour. Others may be tested by substituting successive contour values and locating between which ones the balance changes sign.

glaciers with many tributaries may be subject to different ELAs on different tributaries. Little can be done to solve this problem, but it again emphasises the point that the spatial distribution of ELA is best obtained from medium-sized simply shaped glaciers. There is a similar problem in identifying the relevant top/headwall altitude for multiple tributary glaciers.

7. Validation and statistical decision-making

The validation of such estimates of ELAs by these and other methods (e.g. THAR or AAR) seldom receives sufficient attention. For use on existing glaciers there is some justification for the application of a BR or BI similar to that on an already studied glacier in the same area or similar environment. For former glaciers this a priori knowledge is lacking, indeed there is a serious risk of circular argument. The ratio or index set selected either depends on some view of what the former climate was, or has been taken from the literature without consideration of differences of climate. This climate-dependent choice has a major influence on the outcome of the ELA calculation, but the resulting ELA may then be used to draw inferences about that same climate.

To avoid this error and because we have insufficient knowledge about the Quaternary environment of glaciers, it is better to make results self-validating. This may be done by adopting the principle that a homogeneous group of glaciers i.e. of similar type in an environmentally homogeneous restricted area should react similarly to the climate they experience. Hence their ELAs should be closely similar, differentiated only by such local individual factors as shading by valley-side precipices. In statistical terms the standard deviation of these individual ELAs from the group mean value will be less than that of other possible sets of ELA estimates from their means. Therefore for each input value of the ratio or index set we should calculate the standard deviation of its predictions (or the standard error of the mean), and select the value which has the smallest standard deviation.

Since a suitable group are likely to show some degree of homogeneity of estimated ELA even when the ratio or index set is not the optimum one, the standard deviation of a set of preliminary estimates can be used in selecting the group initially to decide the geographical limits of the group or by excluding any which show abnormally large deviations. To yield statistically significant results such a group should preferably be ten or more. When a sufficiently homogeneous group cannot be defined so as to yield a single value ELA, but the glaciers lie for example in successive positions across a mountain range then an alternative approach is to use their ELAs to define a smooth ELA profile across the range and to calculate their individual deviations from that. A further development from this, where there are numerous glaciers distributed over a considerable area, is to use each ELA as a point value from which a trend surface can be constructed; the trend surface with the lowest standard error is the best. Where sufficient glaciers are available, this may produce a better and more usefully interpretable result than grouping (for details see Kaser and Osmaston, 2002).

Table 4
Validation spread sheet for estimating the standard deviations of ELAs of a group of glaciers for different BRs or BIs, using hypothetical data

ROW	Glacier number/ name	ELA for BR = 1	ELA for BR = 1.5	ELA for BR = 2	ELA for BR = 2.5	ELA for BR = 3	MEAN ELA for BR 1-3
2	1	500	480	460	440	420	460
3	2	450	440	430	420	410	430
4	3	470	460	450	420	400	440
5	4	520	510	500	480	460	494
6	5	530	510	480	470	450	488
7	6	480	470	460	430	410	450
8	7	510	500	490	480	470	490
9	8	540	530	520	500	490	516
10	9	460	450	440	420	400	434
11	10	550	530	470	450	440	488
12							
13	MEAN ELA	501	488	470	451	435	469
14	STANDARD DEVIATION	34.8	32.6	27.9	29.6	31.7	29.9

For these hypothetical data the estimated ELA ranges from 435 to 401 with BRs of 1 to 3. Lacking any a priori means of knowing which is correct, the statistically preferable value with the lowest deviation is 470 with a BR of 2.

A simple but convenient spread sheet for summarising and assessing the results of successive trials of BRs or BI sets is illustrated in Table 4. It is so simple that no detailed instructions for its construction are provided. An alternative graphical approach to assessment was used in Kaser and Osmaston (2002) to assess the optimum value for the AAR ratio for Quaternary glaciers on the Rwenzori Mountains. This graphical approach could easily be adapted for use with other methods of ELA estimation.

8. Security and verification

Any scientific programme, including those for ELA estimation, which involves large numbers of data values should provide means of checking data entry, programming and results and avoiding inadvertent formula changes. This need is emphasised by the problems of the Benn and Gemmell spreadsheet. On the other hand, no obstacle must be put in the way of others who wish to modify the programme. The accompanying programmes have therefore been designed with the following points in mind:

- The operation of the whole programme is completely transparent, all the formulae controlling each stage being readily accessible, explicit and checkable.
- The tabular inputs and outputs of each stage are displayed and can be readily checked visually for consistency, until they are no longer required.
- For the AABR method an automatic internal check is provided by calculating the ELA with BR = 1 by two different methods and displaying the result continuously.
- Sample trial data sets are provided, together with their correct outputs.
- To prevent inadvertent changes to the programme once it has been confirmed that it is operating correctly, the work sheet can be made ‘read only’ with the exception of the cells required for data input.
- To ensure ready and permanent availability of the programme, without dependence on the vagaries of possibly ephemeral web-sites, detailed instructions are provided here for compiling these spreadsheets in MS-Excel. These list the formulae to be entered in each operational cell, which can easily be incorporated without errors by copy and paste methods.

For both THAR and AAR methods, the respective ratios represent an implicit and somewhat arbitrary factor to reflect both the hypsometry and the mass balance ratio/index. However these are condensed and confounded together into a single empirical figure that cannot be clearly derived theoretically from or related to either of these variables. Although the same THAR or AAR ratio may be applied validly to each of a group of glaciers if they are homogeneous in both these characteristics, this is not so if some have significantly different hypsometries. The advantage of the altitude × area methods is that they take full account of the detailed and differing hypsometry of each glacier, and can apply to this a chosen factor reflecting the mass balance regime. Thus they can be validly applied to a group of glaciers with widely differing hypsometries. Since the same data (contour belt areas) are desirable for AAR and are necessary for the Altitude × Area methods, there is seldom any reason to choose to use the former. The only reasons to choose THAR rather than an altitude × area method is if the outline of the present or former glacier has not been mapped, if contour information is not available or if only a quick and easy result is required.

Appendix A. Spread sheet for calculating glacier ELA by AA and AABR methods

In the programming instructions below:

"**Text**" is an instruction to enter permanent text (shown in italics) in the indicated cell.

"**Data**" is an instruction to enter data or text in the indicated cells for some selected glacier; these may be changed for every computation.

"**Calculate**" is an instruction to enter a permanent formula in square brackets into the indicated cells, when first constructing the spread sheet (if a column, enter in one cell and use Copy—Paste for the rest). When the operation of all these has been checked and found to be correct they should be protected as 'read only'.

The programme provides for glaciers which span 20 contour belts or fewer. If there are more, either successive pairs must be combined and the VI doubled, or the programme must be extended.

Format cells to show no decimals except where indicated.

First stage, Area × Altitude Shortcut Method

Preparation

Text. Enter the following headings into successive cells of **Row 1**:

A1. Glacier number or name
B1. Contour vertical interval (VI)
C1. Contours including all crossing glacier
D1. Mean belt altitude
E1. Contour belt area
F1. Mean altitude × Area
G1. Balance Ratio
H1. ELA trial Reference contour(1)
I1. Belt area × Alt above ref. contour(1)
J1. Area × Alt × Balance Ratio for contour(1)

Copy cells H1, I1, J1 to further groups of three cells until AI1, AJ1, AK1, but increasing the (number) by 1 each time.

Procedure

1. **Data (A2)**. Glacier name or number,
2. **Data (B2).** Contour vertical interval (VI) (1 decimal place),
3. **Data (C2).** Altitude of a contour below the lower limits of all of the group of glaciers being studied
4. **Calculate (C3-C21).** Extend series of contours to bottom of column. **[= C2 + B2]**
5. **Data (E2-E21).** Measure the areas of successive contour strips on the glacier and enter. Leave blank any cells without glacier area.
6. **Calculate (E23).** Sum areas **[= SUM(E2:E21)]**.
7. **Calculate (F2-F21).** Multiply the area of each contour strip by its mean altitude **[= D2*E2]**
8. **Calculate (F23).** Sum these products **[= SUM(F2:F21)]**.
9. **Calculate (F25).** Divide this sum by the sum of the areas **[= F23/E23]**.
   This yields the **median area × altitude** and a first estimate of the ELA, which can either be accepted without further computation or used as an internal check on the results of the second stage.
   Second stage, Area × Altitude Method with Balance Ratio adjustment
   This iterative technique estimates the ELA by a different method to the first stage, and adjusts it by weighting the area of the ablation or accumulation zone more heavily than that of the other zone, according to the Balance Ratio, that is the ratio of the slopes of the mass balance/altitude graph in these two zones. Each slope is assumed to be linear and is normally greater in the ablation zone. Initially an exact contour is entered as a trial ELA. Glacier mass balances for this and successive contour altitudes above it are calculated. both for BR = 1 and for any desired BR. The former is used as a continuing check against the ELA calculated by the AA method.
   Procedure
10. **Data (G2).** Balance ratio to be tested. Initially enter 1 here to confirm that the output agrees with that from the built-in check computation with BR = 1; subsequently enter any desired value (1 decimal place).
11. **Data (H2).** Contour altitude for the first trial ELA.; this must be below the expected value of the ELA and the tenth contour above it must be higher than the expected ELA.
12. **Calculate (I2-I21)**. Multiply the contour belt altitude by its mean height above the trial reference altitude **[= E2*(D2-H2)]**
13. **Calculate (I23).** Sum these products **[= SUM(I2:I21)]**.
14. **Calculate (J2-J21).** Multiply the area × Alt products of column I by the BR **[= IF(I2 < 0,I2*G2,I2)]**.
15. **Calculate (J23).** Sum these products **[= SUM(J2:J21)]**.
16. **Calculate (K2).** Step up to the next trial reference altitude **[= H2 + B2]**.
17. **Calculate**. Copy cells I1-I23, J1-J23, K2 to successive groups of three columns to the right until columns **AJ, AK, AL** are filled. Delete contents of column AL
18. **Text.** Enter following lines of text in cells **A24-A29**, left aligned.
    RESULTS
    AA ELA (median alt x area, shortcut method) =
    AA ELA (median altxarea, long method, if exact contour) =

AA ELA (median altxarea, long method, if not exact contour) =
AABR ELA (exact contour ELA) =
AABR ELA (contour plus proportion of belt) =

19. **Calculate (L26).** If balance sum with the trial contour is zero, this first contour is the correct ELA and it is entered here, otherwise left blank. Copy this to cells O26, R26, U26, X26, AA26, AD26, AG26, AJ26. **[= IF(L23 = 0,K2,"")]**
20. **Calculate (L27).** If balance sum with the trial contour is negative, this contour is higher than the ELA, and the ELA lies between this and the previous contour, in the proportions of cells I23 and L23; otherwise left blank. Copy this to cells O27, R27 U27, X27, AA27, AD27, AG27, AJ27. **[= IF(L23 < 0,H2 + (B2*ABS(I23))/(ABS(I23) + ABS(L23)),0)]**
21. **Calculate (M28).** If balance sum with the trial contour is zero, this contour is the correct ELA and it is entered here, otherwise zero. Copy this to cells P26, S26, V26, Y26, AB26, AE26, AH26. **[= IF(M23 = 0,K2,"")]**
22. **Calculate (M29).** If balance sum with the trial contour is negative, this contour is higher than the ELA, and the ELA lies between this and the previous contour, in the proportions of cells J23 and M23; otherwise zero. Copy this to cells O27, R27 U27, X27, AA27, AD27, AG27, AJ27. **[= IF(M23 < 0,INT(H$2 + ($B$2*ABS(J23))/(J23 + ABS(M23))),0)]**
23. **Text (G25).** *Check* = .
24. **Calculate (G26).** Check whether the ELAs estimated by shortcut and long methods are within +/-1 of each other. If so "TRUE" will be shown, otherwise "FALSE". (This does not check for zero as rounding differences may occur) [= AND(OR(1 > (F25-L27),1 > (F25-O27),1 > (F25-R27),1 > (F25-U27),1 > (F25-X27),1 > (F25-AA27),1 > (F25-AD27),1 > (F25-AG27),1 > (F25-AJ27)),OR(-1 < (F25-L27),-1 < (F25-O27),-1 < (F25-R27),-1 < (F25-U27),-1 < (F25-X27),-1 < (F25-AA27),-1 < (F25-AD27),-1 < (F25-AG27),-1 < (F25-AJ27)))]
25. **Text. (A32-A45).** Enter the following text into these six rows, left aligned (or if preferred print separately):
INSTRUCTIONS 1: Spaces are provided for entry of 20 contour belts. Not all need be used, but more can be added if cell programmes are copied also. Initially values from 2A below have been entered.
INSTRUCTIONS 1A: The AA ELA (median altxarea) calculated by the shortcut method is in cell G25. This should always agree with the 1B value.
INSTRUCTIONS 1B: The AA ELA (median altxarea) calculated by the long method (as for AABR with BR = 1) is in the left-most cell in row 26 or 27 which contains a figure greater than 0.
INSTRUCTIONS 1C: The AABR ELA is in the left-most cell in row 28 or 29 which contains a figure greater than 0.
INSTRUCTIONS 2: Check correct operation of the programme with the following sets of sample data; the check cell H 25 should read TRUE in all cases:
INSTRUCTIONS 2A: Set VI (cell B2) = 100, BR (cellH2) = 1, Contour Altitudes (cells C2 to C21) = 0 to 1900, Contour Belt Areas (cells E4 to E10) = 10 in each cell.
Results 2A: All 3 ELAs should be 550.
INSTRUCTIONS 2B: As for 2A but set BR = 2.
Results 2B: Both AA ELAs should be 450 but AABR ELA should be 489.
INSTRUCTIONS 2C: As for 2A but set BR = 1 and add one contour belt alt 1100, area 10.
Results 2C: All three ELAs should be 600.
INSTRUCTIONS 2D: As for 2C but set BR = 2.
Results 2D: Both AA ELAs should be 600 but AABR ELA should be 530.
INSTRUCTIONS 3: To avoid unintentional alterations to formulae, the work sheet is made 'read only' except for the cells in which data have to be entered i.e. glacier name/number (A2-22), contour vertical interval (B2), lowest contour (C2), contour belt areas (E2-E21) , balance ratio (G2), reference contour (H2). To do this click 'Format – Cells – Protection – Unlock' for each cell or group of cells; then 'Tools – Protection – Protect Sheet – Contents – OK'.
26. Carry out the instructions above.

Appendix B. Construction of partial spread sheet for AABI method

This spread sheet only calculates an ELA for one trial contour altitude and must be repeated manually for other trial contours. Alternatively it may be embedded in an iterative programme like that provided for AABR.

Preparation

Enter the following column headings in Row 1:

A1. MODEL GLACIER Mean belt alts.above or below ELA

B1. MODEL GLACIER Balance index (BI) for each belt above or below ELA(1 decimal place).

C1. NEW GLACIER Contour belt area

D1. NEW GLACIER Mean belt altitude

E1. ELA trial Reference contour

F1. Mean belt Alt MINUS Ref. contour

G1. Mass Balance index (BI)

H1. Area x BI

Procedure

1. **Data (Col. A).** Enter mean belt altitudes for model glacier.

2. **Data (Col. B).** Enter Balance index for each contour belt of model glacier.
3. **Data (Col. C).** Enter areas of contour belts for new study glacier.
4. **Data (Col. D).** Enter mean altitude of each contour belt for new study glacier.
5. **Data (E2).** Enter trial ELA ref. contour.
6. **Calculate (Col. F).** Mean belt alt. minus ref. contour **[= D2-E2]**.
7. **Calculate (Col. G).** Mass balance index related to trial ELA **[= IF(F2 = A2,B2,IF(F2 = A3,B3, IF(F2 = A4,B4,IF(F2 = A5,B5,IF(F2 = A6,B6,IF (F2 = A7,B7,IF(F2 = A8,B8,IF(F2 = A9,B9))))))))]**.
8. **Text (G16).** Enter *[Sum]*
9. **Calculate (Col. H).** Product of belt area and BI **[= C2*G2]**
10. **Calculate (H16)**. Sum products **[= SUM(H2:H14)]**
11. Repeat for other ref. contours. The estimated ELA lies between the contours for which this sum changes sign, at an intermediate value proportionate to these sums.
12. Preferably check operation by entering a series of model BIs representing a linear mass balance curve, e.g. −4, −3, −2, −1, 1, 2, 3, 4, (Col. B), and a series of equal contour belt areas (Col. C). This should yield a median altitude ELA.
13. Write-protect cols. F, G, H.

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